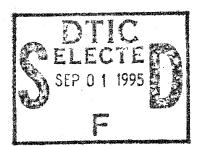
# NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

REPORT No. 585

## SPAN LOAD DISTRIBUTION FOR TAPERED WINGS WITH PARTIAL-SPAN FLAPS



By H. A. PEARSON



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### AERONAUTIC SYMBOLS

#### 1. FUNDAMENTAL AND DERIVED UNITS

		Metric		English					
	Symbol	Unit	Abbrevia- tion	Unit	Abbrevia- tion				
Length Time Force		metersecondweight of 1 kilogram		foot (or mile) second (or hour) weight of 1 pound	ft. (or mi.) sec. (or hr.) lb.				
Power	P V	horsepower (metric) {kilometers per hour meters per second		horsepower miles per hour feet per second	hp. m.p.h. f.p.s.				

### 2. GENERAL SYMPOLS

W.	Weight = mq	
*7 ×	11 015 II 0 - 1100	

g. Standard acceleration of gravity = 9.80665 m/s<sup>2</sup> or 32.1740 ft./sec.<sup>2</sup>

m, Mass  $=\frac{W}{g}$ 

R,

Resultant force

 $I_{i}$  Moment of inertia =  $mk^{2}$ . (Indicate axis of radius of gyration k by proper subscript.)

Coefficient of viscosity

v, Kinematic viscosity

o, Density (mass per unit volume)

Standard density of dry air, 0.12497 kg-m<sup>-4</sup>-s<sup>2</sup> at 15° C. and 760 mm; or 0.002378 lb.-ft.<sup>-4</sup> sec.<sup>2</sup>

Specific weight of "standard" air, 1.2255 kg/m³ or 0.07651 lb./cu.ft.

### 3. AERODYNAMIC SYMBOLS

S,	Area	$i_w$ ,	Angle of setting of wings (relative to thrust
$S_w$ ,	Area of wing		line)
$G_{i}$	Gap	$i_{t_i}$	Angle of stabilizer setting (relative to thrust
b.	Span	*,	line)
	Chord	Q,	Resultant moment
C, 32	Onord	$\Omega$ ,	Resultant angular velocity
$rac{b^2}{S}$	Aspect ratio		•
	M	$ horac{Vl}{\mu}$ ,	Reynolds Number, where $l$ is a linear dimension
V,	True air speed	μ	(e.g., for a model airfoil 3 in. chord, 100
q,	Dynamic pressure $=\frac{1}{2}\rho V^2$		m.p.h. normal pressure at 15° C., the cor-
3.	2°		responding number is 234,000; or for a model
$\mathcal{I}_{i}$	Lift, absolute coefficient $C_L = \frac{L}{aS}$		of 10 cm chord, 40 m.p.s. the corresponding
;	qS		number is 274,000)
D,	Drag, absolute coefficient $C_D = \frac{D}{gS}$	$C_p$ ,	Center-of-pressure coefficient (ratio of distance
ν,	$D_{1}ag$ , absolute coefficient $O_{D} = \overline{qS}$	$U_p$ ,	
$\mathcal{D}$	Profile draw about $D_2$		of c.p. from leading edge to chord length)
$D_{o}$ ,	Profile drag, absolute coefficient $C_{D_o} = \frac{\overline{D_o}}{qS}$	$\alpha$ ,	Angle of attack
70	$T_{i}$ 1.1. $D_{i}$ $m : A_{i} \cap D_{i}$	€,	Angle of downwash
$D_i$ ,	Induced drag, absolute coefficient $C_{D_i} = \frac{D_i}{qS}$	$\alpha_o$ ,	Angle of attack, infinite aspect ratio
-	<u> </u>	$\alpha_i$ ,	Angle of attack, induced
$D_{p_{t}}$	Parasite drag, absolute coefficient $C_{D_{ ho}} = rac{D_{p}}{qS}$	$\alpha_a$ ,	Angle of attack, absolute (measured from zero-
-			lift position)
C,	Cross-wind force, absolute coefficient $C_c = \frac{C}{aS}$	$\gamma_i$	Flight-path angle
-	qs	• •	

### REPORT No. 585

## SPAN LOAD DISTRIBUTION FOR TAPERED WINGS WITH PARTIAL-SPAN FLAPS

By H. A. PEARSON

Langley Memorial Aeronautical Laboratory

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### REPORT No. 585

### SPAN LOAD DISTRIBUTION FOR TAPERED WINGS WITH PARTIAL-SPAN FLAPS

By H. A. Pearson

#### SUMMARY

Tables are given for determining the load distribution of tapered wings with partial-span flaps placed either at the center or at the wing tips. Seventy-two wing-flap combinations, including two aspect ratios, four taper ratios, and nine flap lengths, are included. The distributions for the flapped wing are divided into two parts, one a zero lift distribution due primarily to the flaps and the other an additional lift distribution due to an angle of attack of the wing as a whole.

Comparisons between theoretical and experimental results for wings indicate that the theory may be used to predict the load distribution with sufficient accuracy for structural purposes.

Simple computing forms are included for determining, by the Lotz method, the theoretical loadings for a combination of any wing with any flap. A discussion of the method is given showing: (1) the effect on the load distribution of increasing the number of harmonics for a wing with partial-span flaps; and (2) the effect of increasing the number of points used across the semispan for a wing of unfair plan form.

#### INTRODUCTION

A knowledge of the span load distribution over a wing is important not only from structural considerations but also because certain conclusions regarding the behavior of the wing near the stall may be drawn from it. Indirectly, the span load distribution also influences such items relating to performance as the magnitude of the induced drag, the pitching moment of the entire wing about an aerodynamic center, and the angle of zero lift. Because of the importance of span load distribution, numerous methods for computing it have been proposed but, since they are generally lengthy and complicated, they have been little used in practice.

In reference 1 the span loading was given for linearly tapered wings with rounded tips. The results given therein cover a large range of aspect ratios and taper ratios, but they are for the case of a wing in which there is either no twist or only linear twist. Since most airplanes include some sort of high-lift or drag-increasing device covering only part of the span, the wing with an abrupt twist is of particular interest. These high-

lift devices, when deflected, may be considered as introducing an effective twist that alters the load distribution along the span. As the actual effective twist depends upon possible combinations of wing angle of attack, flap type, flap deflection, flap span, wing plan form, and the variation of the flap-chord ratio along the span, it is apparent that the resulting load distribution depends upon many variables.

The presence of so many variables precludes the possibility of making either sufficiently extensive theoretical or experimental investigations to provide design charts for the general case. The present report therefore covers only the most commonly used series of wings; i. e., linearly tapered wings with rounded tips having chord distributions like those of reference 1 and equipped with partial-span flaps of constant flap-chord ratio. Comparisons are made of the experimental loadings, taken from reference 2, and the theoretical loadings to give an indication of the differences to be expected when the theory is used. Finally, a method for computing the span loading is included so that those interested will be in a position either to estimate from the results given herein the probable loading for similar cases or, if necessary, actually to make the computations.

Although the present report presents only the span loadings, later reports will deal with the effect of the load distribution on performance and on the behavior of the wing near the stall.

#### SYMBOLS

- $b_f$ , flap span.
- $b_w$ , wing span.
- S, wing area.
- A, aspect ratio,  $b_w^2/S$ .
- $\delta_f$ , flap deflection, positive downward.
- V, wind velocity.
- $\rho$ , mass density of air.
- q, dynamic pressure,  $\frac{1}{2}\rho V^2$ .
- w, induced downflow at a section.
- L, lift on wing.
- $C_L$ , wing lift coefficient, L/qS.
- $c_s$ , chord at plane of symmetry.
- c, chord at any section.

 $\alpha_0$ , effective angle of attack of any section.

 $\alpha_a$ , angle of attack of any section referred to its zero-lift direction.

 $\alpha_s$ , angle of attack of section at plane of symmetry referred to its zero-lift direction.

λ, ratio of fictitious tip chord, obtained by extending leading and trailing edges of wing to extreme tip, to the chord at the plane of symmetry.

E, ratio of flap chord to wing chord at any section.

 $\Gamma$ , circulation at a section.

l, section lift (per unit length along span).

 $c_l$ , section lift coefficient, l/qc, perpendicular to wind at infinity.

Subscripts:

0, refers to section lift coefficient perpendicular to local relative wind.

b, refers to basic lift  $(C_L=0)$ .

a, refers to additional lift for any  $C_L$ . a1, refers to additional lift for  $C_L=1.0$ .

 $c_{l_{max}}$ , maximum lift coefficient for any section.

 $\Delta c_i$ , increment in section lift coefficient caused by a flap deflection,  $\delta_f$ .

 $c_{d_i}$ , section induced-drag coefficient.

 $c_{d_0}$ , section profile-drag coefficient.

 $L_a$ , additional-load parameter,  $c_{l_{al}} \frac{cb}{S}$ .

 $L_b$ , basic-load parameter,  $c_{l_b} \frac{cb}{S\Delta c_l}$ .

 $m, \frac{dC_L}{d\alpha}$  of entire wing, per radian.

 $m_0$ ,  $\frac{dc_1}{d\alpha_0}$  of any section, per radian.

 $m_s$ ,  $\frac{dc_l}{d\alpha_0}$  of section at plane of symmetry, per radian.

 $\Delta C_L$ , the part of  $C_L$  at a given wing attitude due to any flap deflection.

 $\Delta C_{L_1}$ , the increment caused by a flap deflection corresponding to a  $\Delta c_I$  of 1.0.

y, variable point along span.

y', fixed point along span.

 $\cos \theta$ ,  $\frac{y}{b_w/2}$  (when  $y=-b_w/2$ ,  $\theta=0$ ; when  $y=b_w/2$ ,  $\theta=\pi$ ).

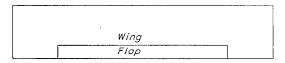
 $A_n, B_n, C_{2n}$ , coefficients in Fourier series.

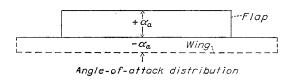
### THEORETICAL RESULTS FOR WINGS WITH FLAPS

According to the assumptions upon which wing theory is based, the distribution of lift over the span is a linear function of the angle of attack at each point of the span. Thus it is permissible to compute separately either a zero lift distribution or a distribution due only to the flaps and later to superpose them on

appropriate distributions due to an angle of attack of the wing with flaps neutral.

Deflecting flaps on an untwisted wing that previously was at zero lift produces the angle of attack and load distributions shown by the solid lines in figure 1. If the angle of attack of the wing without flaps is reduced so that the area under the dashed load curve is equal to that under the solid curve, their addition will result in a zero-lift curve. It can be seen that the load distribution due to the flap alone (solid curve) does not follow





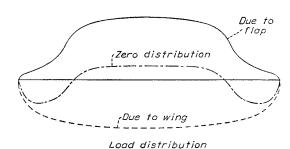


FIGURE 1.—Angle of attack and load distribution for a wing with flaps.

the abrupt angle-of-attack change but, owing to induction, is distributed along the remainder of the span where there is no apparent angle of attack. At these stations there is, however, an effective angle of attack due to the upwash produced by the portion with flaps. Numerically the effective angle of attack at any section is equal to the section  $c_l$  divided by the slope of the section lift curve, or it can be given by

$$\alpha_0 = \alpha_a - \frac{w}{V} \tag{1}$$

In order to determine the theoretical distribution of the forces and angles for a particular case, it is necessary to obtain a solution of the fundamental formula for induced downflow

$$w = \frac{1}{4}\pi \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{d\Gamma}{\frac{dy}{y'-y}}$$
 (2)

The graphical and analytical methods for solving this complicated integral tend to be lengthy and none is exact. In the general case where the wing plan form or angle-of-attack distribution cannot be expressed as simple analytical functions, either the Lotz or Lippisch methods (references 3 and 4) are particularly applicable, although other methods may be used. An adaptation of the Lotz method, which has been used to compute the theoretical load distributions given herein, is given in a later section of this report in a form suitable for routine computation. These load distributions are listed in tables I and II for 72 wing-flap combinations that include two aspect ratios (6 and 10), four taper ratios (1.0, 0.75, 0.50, and 0.25), and nine flap lengths. The flap lengths, expressed as a fraction of the semispan, are:

Flaps at	Flaps at
center	$\operatorname{tip}$
0.233	0.240
. 383	. 351
. 649	. 617
. 760	. 767
1.000	

Table I gives the ordinates of the curves of the additional load distribution at 10 selected spanwise stations in terms of the parameter

$$L_a = c_{l_{a1}} \frac{cb}{S} \tag{3}$$

and table II gives the ordinates for the basic-load distribution in terms of the parameter

$$L_b = c_{I_b} \frac{cb}{S\Delta c_I} \tag{4}$$

The additional-load distribution, given for a wing  $C_L$  of 1.0, is independent of wing twist (flap displacement) and maintains the same form throughout the useful range of the lift curve. The basic distributions are zero lift distributions that depend principally on the wing twist.

The values of  $L_a$  and  $L_b$  were computed by the Lotz method; 10 points across the semispan were used and 10 harmonics of the series were retained. In these computations the slope of the section lift curve was assumed to be equal to 5.67. The odd flap lengths given result from the use of a Fourier series in the solution for the load curves; in the case of a wing with an abrupt twist the discontinuity occurs, mathematically, in the interval including the end of the flap.

Since the parameter  $L_a$  has been given for the convenient wing  $C_L$  of 1.0, the relation between the additional section lift coefficients  $c_{l_a}$  and  $c_{l_{a1}}$  becomes

$$c_{l_a} = C_L c_{l_{a1}} \tag{5}$$

The total lift coefficient at each section is

$$c_l = c_{l_b} + C_L c_{l_a}, \tag{6}$$

and the lift at a section is

$$l = c_i q c \tag{7}$$

In the application of the results given in tables I and II, interpolation will generally be necessary. For structural purposes a linear interpolation between the different variables is probably justified. The results may also be extrapolated with reasonable accuracy to aspect ratios 4 and 12, although values of  $L_a$  may be obtained from reference 1 for aspect ratios from 3 to 20 without the necessity of any extrapolation.

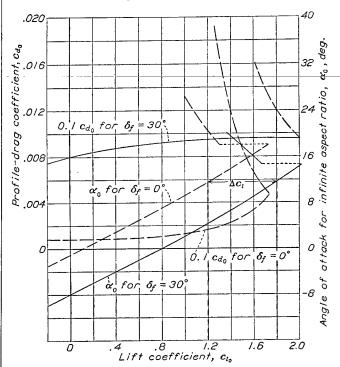


FIGURE 2.—Typical characteristics of a section equipped with a flap.

In order to illustrate the procedure to be followed in the use of the tables, the span loading of a wing with the following characteristics will be found:

$$C_L = 1.72$$
 $\lambda = 0.625$ 
 $A = 6$ 
 $\frac{b_f}{b_w} = 0.383$ .
 $q = 57.5$  pounds per square foot  $E = 0.20$ 
 $\delta_f = 30^\circ$ 

A table, such as table III, is prepared in which the values of the chord at the various stations are first entered, interpolations are made for taper, etc., and the values of  $L_a$  and  $L_b$  from tables I and II are entered in columns 3 and 4, respectively. From  $L_a$ , the values of  $c_{l_{a1}}$  and  $c_{l_a}$  are found by the use of equations (3) and (5) and entered in columns 5 and 6.

Before  $c_{l_b}$  can be found, however, it is necessary to determine from experimental data the value of  $\Delta c_l$  corresponding to the flap-displacement angle of 30°. This increment is generally found by correcting the results of tests made of a finite wing with full-span flaps of proper type and proper flap-chord ratio to

obtain section characteristics. It is assumed that such section characteristics are available (fig. 2); the value of  $\Delta e_l$  to be used may then readily be found. Theoretically,

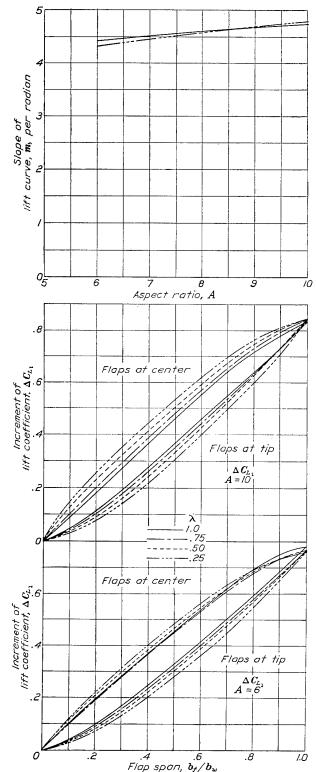


FIGURE 3.—Values of lift increments and lift-curve slopes for wings with partial-span flaps.

the effect of displacing a flap would be to displace the lift curves parallel to each other so that  $\Delta c_i$  would be independent of the effective angle of attack. Experi-

mental results, however, indicate that  $\Delta c_i$  depends on the effective angle of attack and some averaging is thus necessary to determine its value. Since the example is for a high-angle-of-attack condition, the value of  $\Delta c_i$  is arbitrarily taken in this range at an angle corresponding to a  $c_i$  of 1.2 for the plain section. By the use of equation (4) together with a value of  $\Delta c_i$  equal to 0.6, from figure 2, the values of  $c_{i_b}$  are computed and entered in column 7 of table III. The total section  $c_i$  (column 8), from which the load distribution (column 9) is determined, is the sum of columns 6 and 7.

Standard section characteristics for the plain section and the section with a flap are sometimes tabulated instead of being plotted as in figure 2. In such a case the value of  $\Delta c_l$  may be found from the formula

$$\Delta c_l = c_{l_f} \left( 1 - \frac{a_0}{a_{0_f}} \right) + a_0 \left( \alpha_{l_0} - \alpha_{l_{0_f}} \right)$$

where  $\alpha_0$  is the slope of the section lift curve per degree and  $\alpha_{l_0}$  the angle of zero lift measured from the chord line in degrees. The subscript f refers to the characteristics with the flap deflected. If desired, the slopes and angles could also be given in radians.

If the induced-drag distribution corresponding to a given load distribution is specifically required, it may be found by the use of the equation

$$c_{d_i} = c_i \left[ \left( \frac{C_L - \Delta C_{L_1} \Delta c_i}{m} \right) - \frac{c_t}{m_0} \right] \tag{8}$$

which gives the variation of the section induced-drag coefficient over the portion of the span without flaps, and the equation

$$c_{d_i} = c_l \left[ \left( \frac{C_L - \Delta C_{L_1} \Delta c_l}{m} \right) + \left( \frac{\Delta c_l - c_l}{m_0} \right) \right] \tag{9}$$

which holds over the portion of the span with flaps. The increment of wing lift coefficient  $\Delta C_{L_1}$  and the slope of the lift curve of the finite wing m to be used in these equations are given in figure 3 for the series of wings considered in this report. The value of  $\Delta C_{L_1}$  (fig. 3) represents the increase in lift coefficient based on the entire wing area due to a flap deflection corresponding to a  $\Delta c_l$  of 1.0. Figure 4 gives typical distributions of  $c_l \frac{cb}{S}$  and  $c_l$  for various wing-flap combinations corresponding to a  $\Delta c_l$  of 1.0. These distributions are thus directly related to the results given in figure 3.

### COMPARISONS OF EXPERIMENTAL AND THEORETICAL RESULTS

Previous comparisons (reference 5) of experimental and theoretical span loadings for a 2:1 tapered U. S. A. airfoil equipped with partial-span flaps of three different lengths indicated a satisfactory agreement. The first conclusion given in reference 5 is: "A satisfactory determination, for all conditions of test, of the span load distribution for an airfoil equipped with a partial-span split flap may be made by applying the Lotz method of

calculating the aerodynamic characteristics of wings. The increments of load due to the deflection of the flap are computed by the Lotz method and added to the span load distribution for the plain airfoil."

Since the publication of reference 5 additional pressure-distribution tests (reference 2) have been made over a rectangular wing having a 0.6-span constant-chord split flap. The wing used was of Clark Y section with a 20-inch chord and a total span of 120 inches. Some of the span-loading curves taken from reference 2 are compared, in figure 5, with corresponding theoretical curves for a wing with square tips.

section the method will be discussed in more detail and a series of computing forms will be given which, it is believed, will make the computations simpler and more direct than if the method of reference 8 were followed.

Outline of theory.—As is customary in aerodynamic theory, the wing is replaced by a single line vortex whose strength at every section along the span is equal to the circulation  $\Gamma$  at that section. The lift per unit length of span is then

$$dL = \rho V \Gamma dy \tag{10}$$

and the problem is to find  $\Gamma$  for any point on a wing of

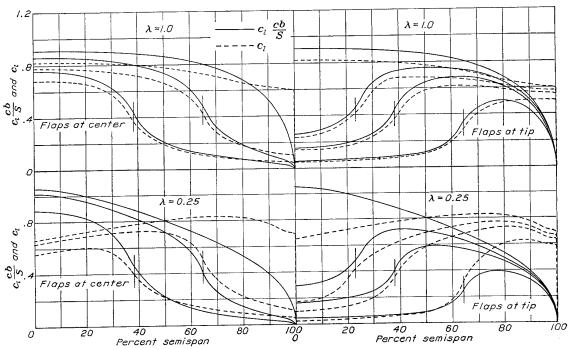


FIGURE 4.—Typical distributions of  $c_i \frac{cb}{S}$  and  $c_i$  due to a flap deflection. (A=6;  $\Delta c_i$ =1.0.)

Figure 6 shows comparisons of computed and experimental values of  $\Delta C_L$  for various flap locations. The experimental values of  $\Delta C_L$  are those given in references 6 and 7 at 8° angle of attack. Reference 6 gives the results of force tests of a rectangular Clark Y wing with partial-span flaps placed at the center and at the wing tips; reference 7 gives similar results for a 5:1 tapered wing. In the comparisons given in figure 6 the experimental results were obtained from tests of wings with straight tips; whereas the computed results are those for wings with rounded tips.

### THE LOTZ METHOD FOR CALCULATING THE AERODYNAMIC CHARACTERISTICS OF WINGS

The following method was proposed in 1931 by Miss Lotz (reference 3), who gave the basic theory involved. Shenstone (reference 8) gave a brief discussion of the method and a simple procedure to be used in obtaining the various constants required in the solution. In this

any shape. The relations between  $\Gamma$ ,  $c_i$ , and  $\alpha_0$  are given by the equations

$$\Gamma = \frac{c_l eV}{2} - \frac{\alpha_0 m_0 eV}{2} \tag{11}$$

where  $\alpha_0 = \alpha_a - w/V$ . Since the induced angle at a particular station y' is

$$\frac{w}{V} = \frac{1}{4\pi V} \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{d\Gamma}{dy} dy \tag{12}$$

the circulation  $\Gamma$  may be expressed by the integral equation

$$\frac{2\Gamma}{m_0 c V} = \alpha_a - \frac{1}{4\pi V} \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{d\Gamma}{dy} dy \tag{13}$$

This equation is to be solved for  $\Gamma$ , and the method used is to replace the circulation by a Fourier series where

 $\Gamma = \frac{c_s m_s V}{2} \Sigma A_n \sin n\theta \tag{14}$ 

and hence

$$\frac{2\Gamma}{m_0 c V} = \frac{m_s}{m_0} \frac{c_s}{c} \Sigma A_n \sin n\theta \tag{15}$$

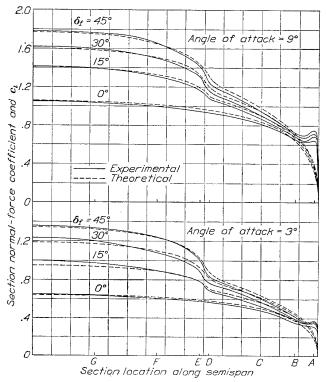


FIGURE 5.—Comparison between experimental and theoretical load distribution for a wing with partial-span flaps. (Data from reference 2;  $\lambda=1.0$ ; A=6; square tips.)

As a result of expressing the circulation by the foregoing series, the induced angle becomes

$$\frac{w}{V} = \frac{c_s m_s}{4b} \sum n A_n \left( \frac{\sin n\theta}{\sin \theta} \right)$$
 (16)

By substitution, equation (13) is transformed into

$$\frac{m_s}{m_0} \frac{c_s}{c} \sin \theta \ \Sigma A_n \sin n\theta = \alpha_a \sin \theta - \frac{c_s m_s}{4b} \Sigma n A_n \sin n\theta \qquad (17)$$

The new feature introduced by Miss Lotz is to replace  $\frac{m_s}{m_0} \frac{c_s}{c} \sin \theta$  and  $\alpha_a \sin \theta$  by the two series  $\Sigma C_{2n} \cos 2n\theta$  and  $\Sigma B_n \sin n\theta$ , respectively. As the coefficients in these series are independent of the load distribution, they may be separately computed, and it is possible to increase the accuracy by taking more terms without

changing the values of the coefficients already computed. When the wing plan form is symmetrical about the center line, the cosine series contains only even values; whereas, if the angle-of-attack distribution is symmetrical, as it is with flaps, only odd values of n are retained. Equation (17), after the foregoing series have been substituted, becomes

$$\Sigma C_{2n} \cos 2n\theta \ \Sigma A_n \sin n\theta + \frac{c_s m_s}{4b} \Sigma n A_n \sin n\theta =$$

$$\Sigma B_n \sin n\theta \tag{18}$$

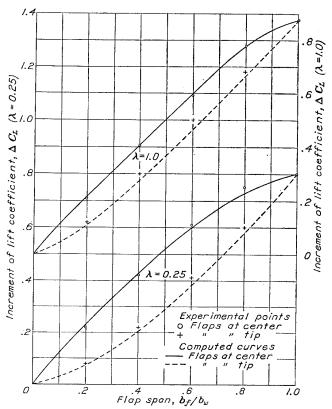


Figure 6.—Comparison of experimental and computed values of  $\Delta C_L$ .

When n is temporarily replaced by the indices k and l, the double series on the left of equation (18) is transformed into

$$\frac{1}{2}\sum_{k}\sum_{l}A_{k} C_{l}[\sin((k+l)\theta+\sin((k-l)\theta)]$$

By the substitution of the foregoing series and considerable rearrangement, equation (18) may be expanded into the following form. In its exact form there are an infinite number of equations and terms. For the purpose of calculation, however, the circulation may be computed at a finite number of points with a finite number of equations.

$$\begin{array}{c} A \\ 2P_{1}A_{1} + (C_{2} - C_{4})A_{3} + (C_{4} - C_{6})A_{5} + (C_{6} - C_{8})A_{7} + (C_{8} - C_{10})A_{1} + (C_{10} - C_{12})A_{11} + (C_{12} - C_{14})A_{15} + (C_{14} - C_{18})A_{15} + (C_{16} - C_{18})A_{17} + (C_{18} - C_{20})A_{19} = 2B_{1} \\ (C_{2} - C_{4})A_{1} + 2P_{3}A_{3} + (C_{2} - C_{8})A_{5} + (C_{4} - C_{10})A_{7} + (C_{6} - C_{12})A_{7} + (C_{8} - C_{14})A_{11} + (C_{10} - C_{16})A_{15} + (C_{12} - C_{18})A_{15} + (C_{14} - C_{20})A_{17} + (C_{16} - C_{22})A_{19} = 2B_{3} \\ (C_{4} - C_{6})A_{1} + (C_{2} - C_{8})A_{2} + 2P_{5}A_{5} + (C_{2} - C_{12})A_{7} + (C_{4} - C_{14})A_{9} + (C_{6} - C_{18})A_{11} + (C_{8} - C_{18})A_{15} + (C_{10} - C_{20})A_{15} + (C_{12} - C_{22})A_{17} + (C_{14} - C_{24})A_{19} = 2B_{5} \\ (C_{6} - C_{8})A_{1} + (C_{4} - C_{10})A_{3} + (C_{2} - C_{12})A_{5} + 2P_{7}A_{7} + (C_{2} - C_{15})A_{9} + (C_{4} - C_{18})A_{11} + (C_{6} - C_{20})A_{13} + (C_{8} - C_{22})A_{15} + (C_{10} - C_{24})A_{17} + (C_{12} - C_{23})A_{19} = 2B_{7} \\ (C_{8} - C_{10})A_{1} + (C_{6} - C_{12})A_{3} + (C_{4} - C_{18})A_{5} + (C_{4} - C_{18})A_{7} + (C_{2} - C_{20})A_{11} + (C_{4} - C_{22})A_{15} + (C_{6} - C_{24})A_{15} + (C_{10} - C_{24})A_{17} + (C_{10} - C_{25})A_{19} = 2B_{9} \\ (C_{10} - C_{12})A_{1} + (C_{8} - C_{16})A_{3} + (C_{6} - C_{16})A_{5} + (C_{4} - C_{18})A_{7} + (C_{2} - C_{20})A_{9} + 2P_{11}A_{11} + (C_{2} - C_{24})A_{15} + (C_{6} - C_{24})A_{15} + (C_{6} - C_{25})A_{17} + (C_{10} - C_{25})A_{19} = 2B_{9} \\ (C_{10} - C_{12})A_{1} + (C_{10} - C_{15})A_{3} + (C_{8} - C_{16})A_{5} + (C_{8} - C_{20})A_{7} + (C_{9} - C_{20})A_{9} + 2P_{11}A_{11} + (C_{2} - C_{24})A_{15} + (C_{4} - C_{25})A_{15} + (C_{6} - C_{25})A_{17} + (C_{6} - C_{25})A_{19} = 2B_{13} \\ (C_{12} - C_{14})A_{1} + (C_{10} - C_{15})A_{3} + (C_{8} - C_{16})A_{5} + (C_{8} - C_{20})A_{7} + (C_{9} - C_{24})A_{11} + (C_{2} - C_{25})A_{15} + (C_{9} - C_{25})A_{17} + (C_{8} - C_{25})A_{19} = 2B_{13} \\ (C_{14} - C_{15})A_{1} + (C_{10} - C_{15})A_{3} + (C_{10} - C_{20})A_{5} + (C_{10} - C_{24})A_{7} + (C_{8} - C_{26})A_$$

$$P_n = C_0 - \frac{1}{2}C_{2n} + n\frac{c_s m_s}{4h}$$

These equations form a system of normal simultaneous equations, and it will be seen later that in the nth equation the unknown  $A_n$  has the greatest coefficient, the others decreasing rather rapidly. Because of this circumstance, the system is most easily solved by a method of successive approximations.

In the first equation, since the value of all the terms is small compared with  $P_1$   $A_1$ , an approximation to  $A_1$ is obtained by assuming all terms except  $A_1$  equal to zero. Then in the next equation, since  $P_3$   $A_3$  is large with respect to all terms except  $(C_2-C_4)$   $A_1$ , which is known, an approximation to  $A_3$  is obtained by assuming the remaining terms equal to zero. Thus by the substitution of the approximated values in the other equations, approximate values of the remaining coefficients are obtained which, when substituted back in the first equation, result in a closer approximation for  $A_1$ . A repetition results in closer approximations for all the coefficients. In this way the process can be carried on until the approximations of the coefficients cease to differ. Usually the second approximation is fairly close, and the third may be considered as exact. (See illustrative example.)

Forms for computing  $B_n$  and  $C_{2n}$  coefficients.—Before the system of simultaneous equations (19) can be solved, the  $B_n$  and  $C_{2n}$  coefficients must be found. Forms for determining these coefficients are given by plates I to IV, inclusive. Plates I and II are for the case when the circulation is to be determined at 10 points across the semispan and plates III and IV are for 20 points.

It is only necessary to tabulate on each of the forms

the values of  $y_n$  and  $y_n'$  and to follow the steps indicated. The values of  $y_n$  are the ordinates for the  $\alpha \sin \theta$  curves taken either every 9° or  $4\frac{1}{2}$ ° (starting with the tip as zero), depending upon whether 10 or 20 points are used. The values of  $y_n'$  are the ordinates of the  $\frac{m_s}{m_0} \frac{c_s}{c} \sin \theta$  curves taken at the same intervals as before. The checks indicated at the bottoms of these forms merely serve as checks of the numerical work performed on that sheet and, if only a few harmonics are to be retained, the arithmetic may be decreased by computing only the coefficients necessary and omitting the checks.

Number of harmonics or points to be retained.—In the series of simultaneous equations given by equation (19) the question naturally arises as to how many equations should be used and how many points across the semispan are required. The system shown is for 10 points, but it may easily be extended to more than 10 points by following the indicated trend. In the case given (equation (19)), the conditions are satisfied at only 10 points when the whole system of equations is solved simultaneously; if the system is cut off, as at A, B, or C, where 4, 5, and 8 harmonics are retained, the circulation may still be found at 10 points but with a greater degree of approximation.

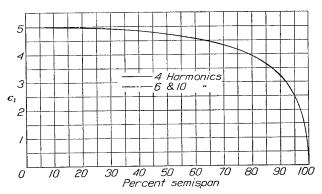


FIGURE 7.—Effect of the number of harmonics on the span of c; distribution of a wing without flaps. (Angle of attack, a, 1 radian.)

As a criterion for gaging the number of harmonics to be retained, the span  $c_l$  distribution has been computed (fig. 7) for an untwisted rectangular wing (straight tips) of aspect ratio 6 at 1 radian angle of attack, using 4, 6, and 10 harmonics. The calculations were repeated (fig. 8) for the same wing with a  $\frac{b_f}{b_w}$ =0.649 flap extending out from the center. The angle of attack for the portion with flap is 1 radian and that of the remainder of the wing is zero. In both cases 10 points have been used across the semispan. The  $A_n$ , or circulation, coefficients from which the distributions of figures 7 and 8 were computed are given in table IV. In figure 9 the distribution has been computed for a wing with double taper. Distributions are given for the case using 10 points

and retaining 4 and 10 harmonics of the series and also for the case with 20 points and 4 harmonics. For convenience the distributions have been computed for an untwisted wing at an angle of attack of 1 radian.

**Example.**—In order to illustrate the method of calculating the wing characteristics, an example for a wing with partial-span flaps is worked through the forms to determine the  $B_n$  and  $C_{2n}$  coefficients. The calculations are made for one of the wing shapes given in this report ( $\lambda$ =0.50, A=10,  $b_f/b_x$ =0.489) at an angle of attack of 1 radian from 0 to 0.489 and 0 from 0.489 to 1.0. The additional types of forms and tables necessary to compute the load distribution for a given case are also included.

Table V is a tabulation of the known geometric quantities of the wing for which the load distribution is desired. Column 1 of this table merely designates the points along the span, the numbers increasing

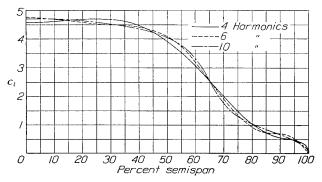


FIGURE 8.—Effect of the number of harmonics on the span  $c_t$  distribution of a wing with flaps. (Angle of attack, 1 radian from 0 to 0.649 and 0 from 0.649 to 1.000.)

numerically from the wing tip; column 5 represents the angle of attack measured from zero lift at the points along the span given in column 2. Where an abrupt twist exists, the discontinuity will fall within the portions of the span given in column 2. The final computations, however, will be for the case of a flap whose end lies halfway between these points. Because of this fact a slight discrepancy in length may occur, which can be reduced by increasing the number of points. In the present case only the distribution due to the flaps is found. In order to obtain a complete determination of the distribution at other flap angles and wing angles, it would also be necessary to find the distribution corresponding to the plain wing. For this case the  $C_{2n}$  coefficients remain unchanged and  $B_1 = \alpha_s$ , all other  $B_n$  values being zero. Column 7 is the slope of the section lift curves along the span which, in this case, is assumed as 5.67. Column 8 is the ratio of the slope of the section at the plane of symmetry to the slopes of the sections at each station. Column 9 is the ratio of the chord at the plane of symmetry to the chord at each section.

The values of columns 6 and 10  $(y_n \text{ and } y_n')$  are then tabulated as shown in table VI and the instructions of plates I and II, or of plates III and IV as in

the present example, are followed until the  $B_n$  and  $C_{2n}$  coefficients are found. If this method were used and only four harmonics were to be retained, it would be only necessary to compute  $B_1$  to  $B_7$  and  $C_0$  to  $C_{14}$  (see  $\Lambda$ , equation (19)); computing the remaining coefficients would be necessary only to obtain the check.

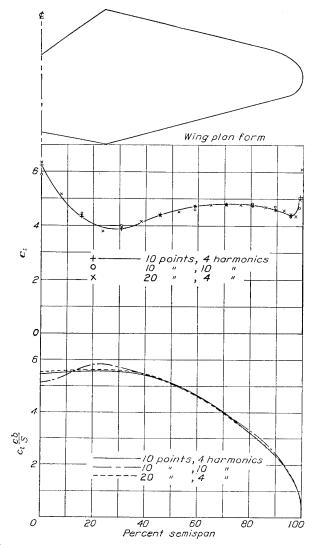


Figure 9.—Effect of the number of points and harmonics on the span  $c_I$  distribution for a wing with double taper.

A calculating form similar to table VII is then prepared. This form, as given, is complete for the case of 10 harmonics irrespective of the number of points. It will be noted that each major horizontal division represents one of the simultaneous equations occurring in equation (19). In column 1 of table VII are given the operations required to obtain the coefficients and in column 2 are tabulated the values of the coefficients, etc., just found. In column 3 (a) are listed the values of the  $A_n$  coefficients when they are known. Since none are known at the start,  $A_1$  is determined as though the others were absent and listed in column 4 (a). The value of  $A_3$  is next approximated in the same way, except that the value of  $A_1$  just found is used as in-

dicated. The same procedure is followed for  $A_5$ ,  $A_7$ , etc., and these values are listed in column 4 (a). After all the  $A_n$ 's have been approximated in this way, they are written in column 3 (b) and the whole process repeated, using the latest approximated value for each coefficient as it appears. It can be seen that the third approximation shows very little change from the second, indicating that a solution has been obtained. If it is desired to use fewer equations and harmonics, the corresponding computing form can be obtained from the present table VII simply by omitting all computations dealing with the higher harmonics. Thus if four harmonics were retained only portions of the form between the braces would be retained and the computations would proceed as before.

It will be noted, in the present example, that, although the  $B_n$  and  $C_{2n}$  coefficients were determined for 20 points, it is not necessary that  $c_l$  be computed for every point to obtain the final load curve. Even though the computations of the load distribution may be somewhat shortened in this manner, the value of  $c_l$  should not be computed at points other than those first selected.

An examination of equation (19) will indicate that, if n harmonics are retained, n values of B and 2n values of C are required. Hence, if it were decided to use 10 harmonics and compute the circulation at 10 points, the  $B_n$  and  $C_{2n}$  values can be determined for 20 points and the process shortened as indicated, or the  $B_n$  coefficients could be determined from plate I and the  $C_{2n}$  coefficients from plate IV.

After the  $A_n$  coefficients have been determined, the  $c_i$  values (in the present case  $c_i = c_{l_a}$ ) are found from

$$c_t = \frac{m_s c_s}{c} \Sigma A_n \sin n\theta \tag{20}$$

These computations for  $c_t$  are given in table VIII for only 10 points. The wing  $C_L$  is found from

$$C_L = \pi A \frac{m_s c_s}{4b} A_1 \tag{21}$$

When this value is known, the distribution at any other  $C_L$  is obtained by direct proportion.

If desired, the induced-drag distribution may also be computed by using the  $A_n$  coefficients

$$c_{d_i} = c_l \frac{m_s c_s}{4b} \sum_{sin \theta} \frac{nA_n \sin n\theta}{\sin \theta}$$
 (22)

as shown in table VIII; however, an easier method would be to compute it at each point from the equation

$$c_{d_i} = c_i \left( \alpha - \frac{c_i}{m_0} \right) \tag{23}$$

### DISCUSSION

Although the computed span-loading curves show a qualitative agreement with the experimental wing curves (fig. 5), it is not so good as might be inferred

from the results for the 2:1 tapered wing of reference 5. In the present comparison, however, the disagreement at the tip may be somewhat discounted since the square tip on a rectangular wing is known to give a high tip load. Comparisons of experimental and theoretical distributions for plain wings have indicated better agreement either as the tip was rounded or as the value of  $\lambda$  was decreased.

Rib-pressure curves taken from reference 2 (fig. 10) show a drop in positive pressure near the trailing edge for a section just beyond the end of the flap. This loss in lift may partly account for the fact that the experimental distributions give sharper breaks than the corresponding computed curves. An improvement in the

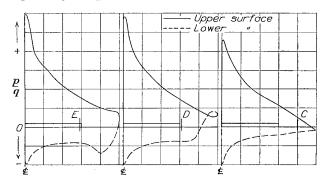


Figure 10.—Rib pressure distribution on a Clark Y wing with a partial-span split flap. (Reference 2;  $\alpha=15^\circ$ ;  $\delta_I=45^\circ$ .)

agreement at the end of the flap may also be obtained by using more points and more harmonics in the series for deriving the theoretical distributions.

For the rest of the span the agreement between the computed and experimental curves would have been slightly improved if jet-boundary corrections had been applied to the data of reference 5. This correction, which varies along the wing span, would effect a better agreement in the present case.

The number of harmonics to be used in computing the span loading depends both on the wing plan form and on the type of wing twist. For wings with a continuous taper and twist, four harmonics may be sufficient (fig. 7); whereas, for wings with either a sharp double taper or a discontinuous twist, it may be necessary to increase the number of harmonics and points (figs. 8 and 9), depending, of course, upon the desired accuracy.

Although the data given herein are intended primarily for structural purposes, they may also be useful in relation to the stalling of tapered wings with flaps. When a partial-span flap is deflected, there is an increase in effective angle of attack and in the value of  $c_{l_{max}}$  for the sections with the flap; whereas, for the sections beyond the flap, the effective angle of attack is theoretically increased without any increase in the value of  $c_{l_{max}}$ . Thus, according to lifting-line theory, the tipstalling tendency of the tapered wing should be augmented by the use of flaps that extend out from the

center, while the center-stalling tendency of the rectangular wing should be increased by flaps at the tips.

Experimental results from reference 2 (fig. 11), however, indicate that the pitching-moment coefficient (or effective camber) of sections considerably beyond the flap are actually increased by a flap deflection. This increase may prevent these outboard sections from stalling as early as would be indicated by the use of lifting-line theory. Furthermore, since theory neglects any transverse flow, any stalling characteristics based upon it may be at best only qualitatively correct. This statement is particularly true of a wing with a partial-span flap, where a relatively large transverse flow exists owing to the abrupt change in lift distribu-

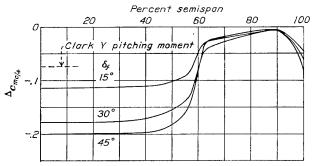


Figure 11.—Increment of pitching moment caused by deflecting a 0.6-span split flap on a rectangular Clark Y wing (reference 2).

tion produced by the flap. Lachmann's tests (reference 9), in which the action of wool tufts was observed, seem to indicate that the transverse flow delays the stall of sections immediately adjacent to the flap, thus causing the initial stalling point to move outward away from the flap end.

In regard to the application of the calculations to structural design, fore-and-aft forces as well as vertical forces must be taken into account. An examination of equation (8) indicates that when  $\Delta C_{L_1} \Delta c_l$  is equal to  $C_L$  (case given by solid lines in fig. 1) the portion of the wing without flaps has its lift vector displaced forward owing to the upflow produced by the flapped part. This forward component may be large enough to cancel the profile drag. Thus, for a wing with flaps at the center, the drag force is concentrated over the flap portion, and there may be an antidrag force over the outer portion of the wing. Hence, in design these conditions should be taken into account in some rational manner.

For structural purposes the  $c_l$  values obtained by use of tables I and II, or by computations, may be considered equal to  $c_{i_0}$ , the lift coefficient perpendicular to the local relative wind. The values of  $c_{l_0}$  and  $c_{d_0}$ , which are perpendicular and parallel to the local relative wind, may then be resolved into either chord and beam or any other directions: the fore-and-aft loads are thus obtained without the explicit use of a section induced drag. The angle that the local relative wind makes with the zero-lift direction is obtained by dividing  $c_{l_0}$  by  $m_0$ . In actual practice a portion of the wing is intercepted by the fuselage so that the actual span load distribution may be modified, depending upon whether or not the fuselage carries its proportionate share of the load. As so few data on fuselage loads are at present available, it may be assumed that for conventional cases the fuselage carries an amount of load equal to the load that would be carried by the wing it displaces.

Langley Memorial Aeronautical Laboratory, National Advisory Committee for Aeronautics, Langley Field, Va., November 21, 1936.

#### REFERENCES

- Anderson, Raymond F.: Determination of the Characteristics of Tapered Wings. T. R. No. 572, N. A. C. A., 1936.
- Wenzinger, Carl J., and Harris, Thomas A.: Pressure Distribution over a Rectangular Airfoil with a Partial-Span Split Flap. T. R. No. 571, N. A. C. A., 1936.
- Lotz, Irmgard: Berechnung der Auftriebsverteilung beliebig geformter Flügel. Z. F. M., vol. 22, no. 7, April 14, 1931, S. 189–195.
- Lippisch, A.: Method for the Determination of the Spanwise Lift Distribution. T. M. No. 778, N. A. C. A., 1935.
- Parsons, John F.: Span Load Distribution on a Tapered Wing as Affected by Partial-Span Flaps from Tests in the Full-Scale Tunnel. Jour. Aero. Sciences, vol. 3, no. 5, March 1936, pp. 161–164.
- Wenzinger, Carl J.: The Effect of Partial-Span Split Flaps on the Aerodynamic Characteristics of a Clark Y Wing. T. N. No. 472, N. A. C. A., 1933.
- Wenzinger, Carl J.: The Effects of Full-Span and Partial-Span Split Flaps on the Aerodynamic Characteristics of a Tapered Wing. T. N. No. 505, N. A. C. A., 1934.
- Shenstone, B. S.: The Lotz Method for Calculating the Aerodynamic Characteristics of Wings. R. A. S. Jour., vol. XXXVIII, no. 281, May 1934, pp. 432-444.
- Lachmann, G. V.: Stalling of Tapered Wings. Flight, vol. XXIX, no. 1410, Jan. 2, 1936, pp. 10-13.

### SPAN LOAD DISTRIBUTION FOR TAPERED WINGS WITH PARTIAL-SPAN FLAPS

### PLATE I.—COMPUTING FORM FOR EVALUATING ANGLE COEFFICIENTS, $B_n$ 10 Points

$y_1$	$y_2$	$y_3$	$y_4$	$y_5$	<b>y</b> 6	$y_7$	<i>y</i> <sub>8</sub>	1/9	$\frac{1}{2}y_{10}$	
$y_1+y_3-y_5$	$-y_7+y_9=r_1$						$y_2 - y_6 +$	$\frac{1}{2}y_{10} = r_2$		
	Multiply					by				
Sin 9=	0. 1564	y <sub>1</sub>		-y <sub>7</sub>			-y <sub>3</sub>		<b>y</b> 9	
Sin 18=	0.3090		$y_2$		$y_6$			$y_6$		$y_2$
Sin 27=	0. 4540	y <sub>3</sub>		$y_1$			- y <sub>9</sub>		$-y_{7}$	
Sin 36=	0.5878		$y_4$		$-y_8$		•	$y_8$		- y <sub>4</sub>
Sin 45=	0.7071	y <sub>5</sub>		<b>y</b> 5		$r_1$	- y <sub>5</sub>		$y_5$	
Sin 54=	0.8090		$y_6$		$y_2$			$y_2$		1/6
Sin 63=	0.8910	y <sub>7</sub>		$-y_9$			$y_1$		$-y_3$	j
Sin 72=	0.9511		<i>y</i> 8		$y_4$			-y4		- y <sub>8</sub>
Sin 81=	0.9877	<b>y</b> 9		<i>y</i> 3			1/2		$y_1$	
Sin 90=	1.0000		½y <sub>10</sub>		$-\frac{1}{2}y_{10}$	r <sub>2</sub>		$-\frac{1}{2}y_{10}$		½y10
Sum co	1. 1									
Sum co	1. 2									
Col. 1+	col. 2		$=5B_{1}$		$=5B_{3}$	$=5B_{5}$		$=5B_{7}$		$=5B_{9}$

Check:  $B_1-B_3+B_5-B_7+B_9-B_{11}+B_{13}-B_{15}+B_{17}-B_{19}=y_{10}$ . Note.—If  $\alpha_a$  is constant along the span,  $B_1=\alpha_s$  and  $B_3$  to  $B_{19}$  are 0.

Col. 1-col. 2....

### PLATE II.—COMPUTING FORM FOR EVALUATING PLAN FORM COEFFICIENTS, $C_{2n}$

 $=5B_{17}$ 

 $=5B_{19}$ 

 $=5B_{15}$ 

 $=5B_{13}$ 

 $=5B_{11}$ 

				10 POIN	NTS				
	1/2 <i>y</i> 10'	$y_1' \\ y_2'$	$y_{8}^{\prime\prime}$	$y_3' \\ y_7'$	$y_4' \\ y_6'$	$y_5$	$v_6$	$v_4$	$v_2$ $v_3$
Sum		$v_1 \\ w_1$	$v_2$ $w_2$	$v_3$ $w_3$	1°4 1°C4	t's	$p_0$	$p_1$ $q_1$	$p_2 = q_2$
				10 $C_0 = p_0 + q_0$ 5 $C_{10} = w_0 - q_0$ 10 $C_{20} = q_0 - q_0$	$p_1 + p_2  w_2 + w_4  q_1 + q_2$				

Multiply	hy														
Sin 18=0.3090	$w_4$		$q_2$	<i>p</i> <sub>2</sub>	$-w_{2}$		$p_1$	$-q_1$							
Sin 36=0.5878		$w_3$				$w_1$									
Sin 54=0.8090	$u_{2}$		$q_1$	$-p_1$	w <sub>4</sub>		$-p_2$	$-q_2$							
Sin 72=0.9511		$w_1$				$w_3$									
Sin 90=1.00000	$w_0$		$q_0$	$p_0$	$w_0$		$p_0$	g <sub>0</sub>							
Sum col. 1															
Sum col. 2			$=5C_4$	$=5C_{16}$			$=5C_8$	$=5C_{12}$							
Col. 1+col. 2	= (	$5C_2$				$=5C_6$									
Col. 1-col. 2	== {	$5C_{18}$				$=5C_{14}$									

Check:  $C_0 + C_2 + C_4 + C_6 + C_8 + C_{10} + C_{12} + C_{14} + C_{16} + C_{18} + C_{20} = 0$ .

### REPORT NO. 585—NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

### PLATE III.—COMPUTING FORM FOR EVALUATING ANGLE COEFFICIENTS, $B_n$

20 POINTS

*y*5  $y_{\hat{n}}$ *y*s  $y_{10}$   $y_{11}$ 1/2//20  $y_1$  $y_2$ **y**3 1/1 1/7  $y_0$  $y_{12}$  $y_{13}$  $y_{14}$  $y_{15}$ y16 7/17 1/18 3/19

 $\begin{array}{l} y_1 + y_7 - y_9 - y_{15} + y_{17} = r_1 \\ y_2 + y_6 - y_{10} - y_{14} + y_{18} = r_2 \\ y_3 + y_5 - y_{11} - y_{13} + y_{19} = r_3 \\ y_4 - y_{12} + \frac{1}{2}y_{20} = r_4 \end{array}$ 

Multiply								by								
Sin 4.5=0.0785	$y_1$		y <sub>13</sub>		y <sub>17</sub>	2	<b>/</b> 9	$-y_{11}$		y <sub>3</sub>			y7		$-y_{19}$	
Sin 9.0=0.1564		1/2	y <sub>14</sub>		-	y6	$y_{18}$		$y_{18}$		$-y_{6}$			$-y_{14}$	Ì	$y_2$
Sin 13.5=0. 2334	$y_3$		$y_1$		$-y_{11}$	1	/13	- y7		y <sub>9</sub>			y19		$y_{17}$	
Sin 18.0=0.3090		$y_4$	y <sub>12</sub>			y <sub>12</sub>	$y_4$		$-y_4$		$-y_{12}$			$-y_{12}$		$-y_{4}$
Sin 22.5=0.3827	$y_5$		y <sub>15</sub>	$r_{\rm I}$	<b>y</b> 5	-1	/5	y <sub>15</sub>		$y_{15}$		$-r_3$	<i>y</i> 5		$-y_{15}$	
Sin 27.0=0.4540		$y_6$	y <sub>2</sub>		_	$y_{18}$	$-y_{14}$		$-y_{14}$		$-y_{18}$	- <b>-</b>		$y_2$		y <sub>6</sub>
Sin 31.5=0.5225	$y_7$		$y_{11}$	<b></b>	$y_1$	-2	117	$y_3$		y <sub>19</sub>			$-y_9$		y <sub>13</sub>	
Sin 36.0=0.5878		$y_8$	y <sub>16</sub>			y16	$-y_{8}$		$y_8$		$-y_{16}$			$y_{16}$		$-y_8$
Sin 40.5=0.6494	$y_9$		$y_3$		$-y_7$	ı	/1	$-y_{19}$		$y_{13}$			$-y_{17}$		$-y_{11}$	
Sin 45.0=0.7071		$y_{i0}$	<b>y</b> 10	r <sub>2</sub>	-	$y_{10}$	$y_{10}$		$y_{10}$		$-y_{10}$	$r_2$		$y_{10}$		$y_{10}$
Sin 49.5=0.7604	$y_{11}$		$-y_{17}$		y <sub>13</sub>	l v	/19	$y_1$		<i>y</i> 7			$-y_3$		$y_9$	
Sin 54.0=0.8090		$y_{12}$	<i>y</i> 4			$y_4$	<b>y</b> 12		$-y_{12}$		$-y_4$			$-y_4$		$-y_{12}$
Sin 58.5=0.8526	$y_{13}$		<b>y</b> 0		$-y_{19}$	y	/3	y <sub>17</sub>		$y_1$			y <sub>11</sub>		-y7	
Sin 63.0=0.8910		$y_{14}$	$-y_{18}$			y <sub>2</sub>	$-y_6$		$-y_5$		$y_2$			$-y_{18}$		y <sub>14</sub>
Sin 67.5=0.9239	$y_{15}$		<b>y</b> 5	r <sub>3</sub>	$y_{15}$	-y	15	$-y_5$		y <sub>5</sub>		$r_1$	y <sub>15</sub>		<b>y</b> 5	
Sin 72.0=0.9511		$y_{16}$	Уs		-	ys	$-y_{16}$		$y_{16}$		1/8			$-y_{8}$		$-y_{16}$
Sin 76.5=0.9724	$y_{17}$		$-y_{10}$		$-y_9$	-y	17	$-y_{13}$		$-y_{11}$			$y_1$	į	$-y_{3}$	
Sin 81.0=0.9877		<b>y</b> 18	$y_6$			y <sub>14</sub>	$y_2$		$y_2$		$y_{14}$			$y_6$		$y_{18}$
Sin 85.5=0.9969	$y_{19}$		<b>y</b> 7		<b>y</b> 3	y	111	<b>y</b> 9		$-y_{17}$			$-y_{13}$		<b>y</b> 1	
Sin 90.0=1.0000		$\frac{1}{2}y_{20}$	$-\frac{1}{2}y_{20}$	74	-1/2	<b>y</b> 20	1/2y20		$-\frac{1}{2}y_{20}$		1/21/20	$-r_4$		1/21/20		1/2 <b>/</b> /20
Sum col. 1																
Sum col. 2																
Col. 1+col. 2		10 <i>B</i> i	$10B_3$	$10B_{5}$	107	$B_7$	10 <i>B</i> <sub>2</sub>		10 <i>B</i> <sub>I1</sub>		$10B_{13}$	10 <i>B</i> <sub>15</sub>		$10B_{17}$	1	$0B_{19}$
Col. 1-col. 2		$10B_{30}$	$10B_{37}$	$10B_{35}$	(0)	B33	$10B_{31}$		10829		10 <i>B</i> 27	$10B_{25}$		$10B_{23}$	1	$0B_{2!}$

Check:  $B_1 - B_3 + B_5 - B_7 + B_9 - \dots + \dots B_{39} = y_{20}$ .

### SPAN LOAD DISTRIBUTION FOR TAPERED WINGS WITH PARTIAL-SPAN FLAPS

### PLATE IV.—COMPUTING FORM FOR EVALUATING PLAN FORM COEFFICIENTS, C2n

### 20 POINTS

	1/2y <sub>20</sub> '	$y_1' \\ y_1_9'$	$y_2' \\ y_1$ 8'	$y_{17}^{\prime\prime}, y_{17}^{\prime\prime}$	$y_4'$ $y_{16}'$	y5' y15'	y6' y <sub>14</sub> '	y1' y13'	$y_{12}'$	y <sub>9</sub> ' y <sub>11</sub> '	y <sub>10</sub> '
Sum Difference	$v_0 = v_0$	$v_1 \\ w_1$	$v_2 \\ w_2$	$v_3$ $w_3$	$v_4$ $w_4$	$v_5$ $w_5$	$v_6$ $w_6$	$v_7 \\ w_7$	$v_8$	$v_9$	$v_{10}$
	$v_0$ $v_{10}$	$v_1 \\ v_9$	$egin{array}{c} v_2 \ v_8 \end{array}$	$v_3 \\ v_7$	$v_4 \\ v_6$	$v_5$		$egin{array}{c} p_0 \ p_5 \end{array}$	$p_1 \\ p_4$	$egin{array}{c} oldsymbol{p_2} \ oldsymbol{p_3} \end{array}$	
Sum Difference	po	$p_1 = q_1$	$q_2$	$\frac{p_3}{q_3}$	$q_4$	$p_5$		70 80	$r_1$ $s_1$	$r_2$ $s_2$	

 $\begin{array}{l} 20 \ C_0 = p_0 + p_1 + p_2 + p_3 + p_4 + p_5 \\ 10 \ C_{10} = \sqrt{2}(w_1 - w_3 - w_5 + w_7 + w_9) + w_0 - w_4 + w_8 \\ 10 \ C_{20} = g_0 - g_1 + g_4 \\ 10 \ C_{30} = \sqrt{2}(-w_1 + w_3 + w_5 - w_7 - w_9) + w_0 - w_4 + w_8 \\ 20 \ C_{40} = p_0 - p_1 + p_2 - p_3 + p_4 - p_5 \end{array}$ 

Multiply							by					٠		
Sin 9=0.1564		$w_9$			$w_3$			$w_7$		$w_1$			   <b>-</b>	
Sin 18=0.3090	$w_8$		<i>q</i> <sub>4</sub>	$-w_{4}$		$-q_2$	$-w_{4}$		$w_8$		82	$r_1$	-81	r <sub>2</sub>
Sin 27=0.4540		$w_7$			$-w_9$			$w_1$		$-w_3$				
Sin 36=0.5878	$w_6$		$q_3$	$w_2$		$q_1$	$-w_2$		$-w_6$		<b></b>			
Sin 45=9.7071		$w_5$			$-w_5$			$w_5$		$w_5$				
Sin 54=0.8090	$w_4$		<i>q</i> <sub>2</sub>	$-w_8$		q <sub>4</sub>	$-w_8$		$w_4$		$s_1$	$-r_2$	-82	-r
Sin 63=0.8910		$w_3$			$w_{\mathfrak{l}}$			$-w_9$		$-w_{7}$				
Sin 72=0.9511	$w_2$		$q_1$	$-w_{5}$		$-q_3$	$w_{\mathfrak{b}}$		$-w_2$				<b>-</b>	
Sin 81=0.9877		$w_1$			$-w_7$			$-w_3$		$w_{\mathfrak{g}}$				
Sin 90=1.0000	$w_0$		$q_0$	$w_0$		$q_0$	$w_0$		$w_0$		80		80	r
Sum col. 1											$10C_{8}$	$10C_{16}$	$10C_{24}$	10C3
Sum col. 2											<b>-</b>		- <b>-</b>	
Col. 1+col. 2		$10C_2$	10C4		$10C_6$	$10C_{12}$		$10C_{14}$		$10C_{18}$				
Col. 1-col. 2		$10C_{38}$	10C <sub>36</sub>		$10C_{34}$	$10C_{28}$		$10C_{26}$		$10C_{22}$				

Check:  $C_0+C_2+c_4....C_{40}=0$ .

### TABLE I.—VALUES OF $L_a$ FOR TAPERED WINGS WITH ROUNDED TIPS

	A=6																A=10					
	$\lambda \frac{y/b}{2}$	0	0. 15	0.30	0.45	0.60	0.70	0.80	0. 90	0.95	0. 975	0	0. 15	0.30	0. 45	0.60	0.70	0.80	0. 90	0.95	0. 975	$\frac{y/b}{2}$
	1.00 .75 .50 .25	1. 164 1. 217 1. 291 1. 392	1. 163 1. 204 1. 263 1. 349	1. 144 1. 167 1. 191 1. 243	1. 115 1. 112 1. 107 1. 118	1. 050 1. 026 . 995 . 954	0. 987 . 953 . 908 . 841	0. 870 . 840 . 789 . 709	0. 669 . 648 . 607 . 521	0. 485 . 468 . 447 . 386	0.358 .340 .319 .286	1. 116 1. 194 1. 292 1. 424	1. 111 1. 179 1. 257 1. 368	1. 106 1. 140 1. 184 1. 247	1. 090 1. 089 1. 093 1. 104	1. 052 1. 020 . 982 . 940	1. 011 . 964 . 903 . 823	0. 929 . 875 . 800 . 695	0. 757 . 710 . 648 . 528	0. 572 . 536 . 492 . 407	0. 433 . 396 . 367 . 308	1. 00 . 75 . 50 . 25

TABLE II.—VALUES OF L, FOR TAPERED WINGS WITH ROUNDED TIPS

[Valves of  $L_b$  given for flaps at center. Reverse signs when using for flaps at tips]

						Ţ .			
	Flaps at tip								Constant of the Constant of th
	9/b 2 hr 5 bw		0.767 .617 .351		0. 767 . 617 . 351		0.767 .617 .351		0.767 . 617 . 351
	0.975		-0.077 126 210 227		-0.077 125 203 223		-0.077 124 192 207		-0.075 113 173 183
	0.95		-0.107 175 280 307		-0.107 174 275 298		-0.107 -171 -260 -281		-0.100 154 228
	0.90		-0.140 231 360 386		-0.141 229 352 373		-0.140 224 332 352		-0.129 200 287 300
	0.80		-0.170 277 402 340		-0.174 274 391 327		-0.169 266 375		-0.161 247 335 278
	0.70		-0.181 283 317		-0.181 283 314 001		-0.182 281 308 004		-0.183 276 266 009
A=10	09.00		-0.173 266 033 . 081		-0.177 -272 .027 .027		-0.183 277 .021		-0.191 -285 .004
	0.45		-0.146 145 178		-0.155 -156 -169 .169		-0.167 170 .159		-0.186 193 .089
	0.30		-0.023 .305 .211		-0.027 .301 .206 .133		-0.038 .297 .198		-0.051 . 285 . 183 . 112
	0.15		0. 415 . 405 . 229 . 147		0. 423 . 414 . 230 . 147		0. 436 - 424 : 226 . 141		0.445 .425 .213 .126
	0		0.487 .423 .232 .150	75	0.509 .433 .235	20	0. 534 . 447 . 234 . 148	25	0.558 .454 .223 .133
	0.975	$\lambda = 1.00$	-0.061 100 157 171	λ=0.75	-0.059 096 149 155	$\lambda = 0.50$	-0.059 096 146 150	λ=0.25	-0.058 090 131 139
	0.95		-0.085 139 209		-0.083 134 204 213		-0.082 134 197 206		-0.079 124 179 186
	0.90		-0.116 185 275 289		-0.113 180 266 273		-0.111 176 256 263		-0.106 164 234 237
	0.80		-0.145 227 313 255		-0.143 222 305 248		-0.141 218 295 240		-0.133 205 272 221
4=6	0.70		-0.155 236 251 013		-0.154 233 245 021		-0.153 230 240 022		-0.151 226 231 023
,	0.60		-0.147 221 .006		-0.148 218 .006		-0.151 221 .002 .043		-0.136   -0.154   -2.25   -0.164   -0.006   -0
	0.45		-0,117 -,112 -,133 ,093		-0.120 114 .125		-0.124 123 .118		-0.136 134 .108 .070
	0.30		-0.002 .238 .169		-0.004 . 236 . 164 . 104		-0.008 .227 .159		-0.016 . 222 . 149 . 092
	0.15		0.332 .334 .189		0.341 .333 .186		0.340 .335 .182		0.342 .336 .175
	0		0.400 .351 .192 .110		0.406 .353 .192 .118		0.420 .360 .191		0. 425 . 361 . 186 . 109
	br   V/b		0.233 .383 .649 .760		0.233 . 383 . 649 . 760		0.233 .383 .649 .760		0.233 .383 .649 .760
	Flaps at center								

### TABLE III.—CALCULATION OF LIFT DISTRIBUTION FOR ILLUSTRATIVE EXAMPLE

### TABLE IV.—CIRCULATION COEFFICIENTS

[A, 6,  $\lambda$ , 1.0,  $b_f/b_w$ , 0.649]

[ 4 C. L /h	0.000	C 988 7 00	ft . h . 10 !	+ · 1 0 695•	C. 179	$\Delta c_1, 0.60; q, 3$	57 51
$-1A.6:bdb_{\nu}$	0.383:	S. 266.7 SQ.	. IT.: 0. 40 I	t.: A. U.025:	CL. 1.72,	$\Delta c_1, 0.00, q$	77.0]

1	2	3	4	5	6	7	8	9
Station $y/\frac{b}{2}$	Chord (ft.)	$L_{\mathfrak{a}}$	$L_b$	Cl al	CI a	cı <sub>b</sub>	C1	(lb.)
0 .15 .30 .45 .60 .70 .80 .95 .975	8. 500 8. 022 7. 544 7. 066 6. 588 6. 269 5. 820 4. 825 3. 655 2. 635	1. 254 1. 233 1. 179 1. 110 1. 010 . 930 . 815 . 628 . 457 . 330	0.356 .334 .232 118 220 232 220 179 134 097	0. 984 1. 025 1. 043 1. 047 1. 022 . 989 . 934 . 868 . 835 . 835	1. 693 1. 762 1. 794 1. 801 1. 758 1. 700 1. 607 1. 492 1. 435 1. 435	0. 168 . 167 . 123 067 134 148 151 148 147 147	1. 861 1. 929 1. 917 1. 734 1. 624 1. 552 1. 456 1. 344 1. 288 1. 288	91. 0 89. 0 83. 1 70. 5 61. 6 56. 0 48. 7 37. 3 27. 1 19. 5

	No:	flaps		Flaps at center						
Coeffi- cient	4 har- monics retained	6 har- monics retained	10 har- monics retained	Coeffi- cient	4 har- monics retained	6 har- monics retained	10 har- monics retained			
A <sub>1</sub> A <sub>3</sub> A <sub>5</sub> A <sub>7</sub> A <sub>9</sub> A <sub>11</sub> A <sub>13</sub> A <sub>15</sub> A <sub>17</sub>	0. 9280 . 1158 . 0251 . 0069	0. 9290 . 1160 . 0251 . 0072 . 0026 . 0011	0. 9290 .1161 .0251 .0073 .0026 .0011 .0005 .0003 .0002	A1 A3 A5 A7 A0 A11 A13 A18 A17	0. 6682 1825 0298 . 0588	0. 6684 1826 0301 . 0585 . 0017 0286	0. 6682 1826 0300 0586 0019 0281 0058 0083 0104			

### TABLE V.—GEOMETRIC CHARACTERISTICS OF WING USED IN EXAMPLE

1	2	3	4	5	6	7	8	9	10
A	Fraction of semispan	θ (deg.)	sin θ	α (rad.)	α sin θ	$m_0$ for $b=\infty$	$\frac{m_s}{m_0}$	c₄ c	$\frac{m_s c_s}{m_0 c} \sin \theta$
20 19 18 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2	0 .0785 .1564 .2334 .3090 .3827 .4540 .5225 .5878 .6494 .7071 .7664 .8090 .8526 .8910 .9239 .9511 .9724 .9877 .9877	90 85.5 81 76.5 72 67.5 63 58.5 54 49.5 40.5 31.5 22.5 18 13.5 9	1, 0000 9969 9877 9877 9724 9511 9239 8910 8526 8090 7604 7071 6494 5878 5225 4540 3827 3090 2334 1564 0785	1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0	1. 0000 . 9969 . 9877 . 9724 . 9511 . 9239 . 8910 0 0 0 0 0 0 0 0 0 0	5. 67 5. 67	1. 0 1. 0 1. 0 1. 0 1. 0 1. 0 1. 0 1. 0	1. 0000 1. 0400 1. 0848 1. 1295 1. 1827 1. 2386 1. 2987 1. 3555 1. 4162 1. 4785 1. 5469 1. 6100 1. 6793 1. 7415 1. 8219 1. 9605 2. 2504 2. 8280 4. 0742 7. 9740	1. 0000 1. 0370 1. 0717 1. 1030 1. 1252 1. 1445 1. 1530 1. 1560 1. 1455 1. 1245 1. 0939 1. 0458 9869 9100 8272 7500 6963 6600 6372 6260

#### REPORT NO. 585—NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

### TABLE VI.—COMPUTATION OF ANGLE COEFFICIENTS, $B_n$

0.50000.8910

 $\begin{array}{l} r_1\!=\!0\!+\!0\!-\!0\!-\!0.9239\!+\!0.9724\!=\!0.0485\\ r_2\!=\!0\!+\!0\!-\!0\!-\!0.8910\!+\!0.9877\!=\!.0967\\ r_3\!=\!0\!+\!0\!-\!0\!-\!0&+0.9969\!=\!.9969\\ r_4\!=\!0\!-\!0\!+\!0.5000&=.5000 \end{array}$ 

Multiply		by														
Sin 4.5=0.0785_	0	0		0.0763	0	0	0		0	0.0783						
Sin 9 = 0.1564 Sin 13.5=0.2334	0	-0.1394		0	0.1545	0.1545	0		-0.1394	0						
Sin 18 = 0.3090	0 0	0		0 0	0 0	0 0	0 0		0.2327	.2270						
Sin 22.5=0.3827	0	-0.3536	0.0186	0	0	0.3536	0.3536	-0.3815	0	3536						
$\sin 27 = 0.4540$	0	0.0000	0.0103	-0.4484	4045	4045	-0.4484	-0.3618	0	aaa0 0						
Sin 31.5=0.5225	0	0		0 0,1101	-0.5081	0 .1010	.5209		0	l 0						
$\sin 36 = 0.5878$ .	0	— .5591		.5591	0	0	5591		.5591	0						
Sin 40.5=0.6494	0	0		0	0	6474	0	ĺ	6315	0						
$\sin 45 = 0.7071$	0	0	0.0684	0	0	0	0	0.0684	0	0						
Sin 49.5=0.7604. Sin 54 = 0.8090.	0 0	7394 0	i	0 0	.7580	0 0	0		0	0						
$\sin 58.5 = 0.8526$	0	0		8500	0	.8291	0		0	0						
$\sin 63 = 0.8910$	.7939	8800		5500	0	.0291	0	1	8800	0.7939						
Sin 67.5=0.9239	.8536	0	.9210	.8536	8536	1 0	0	.0448	.8536	0.1939						
$\sin 72 = 0.9511$	.9046	0		0	9046	.9046	0	10110	0	9046						
Sin 76.5=0.9724_	.9456	9694		U	0	0	0		0	0						
$\sin 81 = 0.9877$	.9756	0	i	.8800	. 0	0	.8800		0	.9756						
Sin 85.5=0.9969	.9938	0	5000	0	0	0	9694		(	0						
Sin 90 =1.0000.	.5000	5000	.5000	5000	.5000	5000	.5000	5000	.5000	5000						
Sum col. 1	2.7930	-2.0624	0.9396	0.0799	-0.6037	0.5353	-0.0949	-0.3367	0.4548	-0.2049						
Sum col. 2	3,1741	-2.0024	.5684	.4907	6546	.1546	.3725	4316	.0397	-0.2049						
Col. 1+col. 2		$-4.1409 = 10B_3$		$0.5706 = 10B_7$	$-1.2583 = 10B_9$		$0.2776 = 10B_{13}$	$-0.7683 = 10B_{15}$		$0.1600 = 10B_{19}$						
Col. 1-col. 2	$3811 = 10B_{39}$	$.0161 = 10B_{37}$	$.3712 = 10B_{35}$	$4108 = 10B_{33}$	$.0509 = 10B_{31}$		$4674 = 10B_{27}$	$.0949 = 10B_{25}$		$5698 = 10B_{21}$						
			]				1	101525	101523	.0000-101021						

Check:

 $\begin{array}{l} : \\ B_1 - B_3 + B_5 - B_7 + B_9 - B_{11} + B_{13} - B_{15} + B_{17} - B_{19} + B_{21} - B_{23} + B_{25} - B_{27} + B_{29} - B_{31} + B_{33} - B_{35} + B_{37} - B_{19} = \mathbb{F}_{29} \\ 0.5967 + 0.4141 + 0.1508 - 0.0571 - 0.1258 - 0.0690 + 0.0278 + 0.0768 + 0.0495 - 0.0160 - 0.0570 - 0.0415 + 0.0095 + 0.0467 + 0.0381 - 0.0051 - 0.0411 - 0.0371 + 0.0016 + 0.0381 = 1.0000. \end{array}$ 

### COMPUTION OF PLAN-FORM COEFFICIENTS, C2n

	0.5000	0, 6260 1, 0370	0.6372 $1.0717$	0.6600 1.1030	0. 6953 1. 1252	0.7500 1.1445	0.8272 1.1530	0.9100 1.1560	0.9869 1.1455	1, 0458 1, 1245	1.0939
Sum Difference	0. 5000 5000	1.6630 4110	1. 7089 —. 4345	1.7630 4430	1.8205 4299	1.8945 —.3945	1. 9802 3258	2. 0660 2460	2. 1324 1586	2. 1703 0787	1, 0939 1, 0939
		0.5000 1.0939	1.6630 2.1703	1.7089 2.1324	1.7630 2.0660	1.8205 1.9802	1.8945	1. 5939 1. 8945	3.8333 3.8007	3.8413 3.8290	
Sum Differen	ice	1. 5939 5939	3.8333 5073	3.8413 4235	3.8290 3030	3.8007 1597	1.8945	3. 4884 3006	7.6340 .0326	7.6703 .0123	

 $\begin{array}{l} 20C_0 = 1.5939 + 3.8333 + 3.8413 + 3.8290 + 3.8007 + 1.8945 = 18.7927 \\ 10C_{10} = 1.4142(-0.4110 + 0.4430 + 0.3945 - 0.2460 - 0.0787) - 0.5000 + 0.4299 - 0.1586 = -0.0847 \\ 10C_{20} = -0.5939 + 0.4255 - 0.1597 = -0.3301 \\ 10C_{30} = -1.442(0.4110 - 0.4430 - 0.3945 + 0.2460 + 0.0787) - 0.5000 + 0.4299 - 0.1586 = -0.3727 \\ 20C_{40} = 1.5939 - 3.8333 + 3.8413 - 3.8290 + 3.8007 - 1.8945 = -0.3209 \end{array}$ 

Multiply		by														
Sin 9=0.1564 Sin 18=0.3090 Sin 27=0.4540 Sin 36=0.5878	-0.0490 1117	-0.0493 -0.1781	-0.0693 0.1328 .0357	0. 1309 0. 2982	-0.0385 0.1328 1866	-0.0490	0.0038	2. 3589	-0.0101	2. 3701						
Sin 45=0.7071 Sin 54=0.8090 Sin 63=0.8910 Sin 72=0.9511	2790 3478 4133	-0.1781 3426 4825	2554 . 2790 . 1283 3662 . 3099	. 1292		. 1915 2790 3478 . 2192 . 4133	. 0264	6. 2053	0100	<b>-</b> 6. <b>17</b> 59						
Sin 81=0.9877. Sin 90=1.0000.		5939	. 2430 5000	5939	5000	5000 0777	3006	3. 4884	3006	3.4884						
	-1. 2036	$\begin{array}{c}9858 \\6606 \\ -1,6464 = 10C_4 \\3252 = 10C_{36} \end{array}$		$\begin{array}{c}3338 \\0100 \\3438 = 10 C_{12} \\3238 = 10 C_{28} \end{array}$	$2898 = 10 C_{14}$	$\begin{array}{c}2920 \\0007 \\2927 = 10 C_{18} \\2913 = 10 C_{22} \end{array}$	2704 10 C <sub>8</sub>	$3580$ $10 C_{16}$	3207 10 C <sub>24</sub>	3174 10 C <sub>32</sub>						

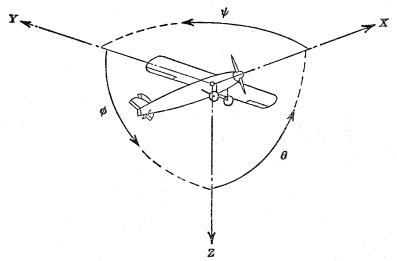
### SPAN LOAD DISTRIBUTION FOR TAPERED WINGS WITH PARTIAL-SPAN FLAPS

### TABLE VII.—SOLUTION OF $A_n$ COEFFICIENTS

1	2	3 (a)	3 (b)	3 (c)	4 (a)	4 (b)	4 (c)	1	2	3 (a)	3 (b)	3 (c)	4 (a)	4 (b)	4 (e)
$\begin{cases} (C_2-C_1)A_3 \\ (C_4-C_2)A_5 \\ (C_6-C_2)A_5 \\ (C_6-C_2)A_7 \\ (C_1-C_1)A_1 \\ (C_1-C_1)A_{11} \\ (C_1-C_1)A_{15} \\ (C_1-C_1)A_1 \\ (C_1-C_1)A_$	. 0208 0185 . 0259 0054 . 0068		0201 . 0117 . 0032	. 0169 0400 . 0189 . 0087 0197	0 0 . 5967	0. 0271 0131 .0004 .0008 .0005 0 0001 0001 0 .0155 .0077 .5890	0130 0004 0007 0005 0 0001 0001 0 0147 0073 5894	$ \begin{array}{c} (C_{10}-C_{12})A_1, \\ (C_3-C_{14})A_3, \\ (C_6-C_{16})A_5, \\ (C_4-C_{16})A_7, \\ (C_2-C_{20})A_9, \\ (C_2-C_{24})A_{15}, \\ (C_4-C_{26})A_{15}, \\ (C_6-C_{28})A_{17}, \\ (C_8-C_{20})A_{19}, \\ (C_8$	0. 0259 . 0020 . 0296 1353 2375 2384 1349 . 0262 . 0103  . 0690 /3. 0654	2563 . 0824 . 0182 0411	0. 4650 2487 . 0823 . 0169 0400 . 0092 0201 . 0117 . 0032	0. 4653 2488 .0823 .0169 0400 .0087 0197 .0118 .0032	0.0122 0005 .0024 0025 .0098  .0214 .0107 .0583 .0190	0. 0120 0005 . 0024 0023 . 0095 0022 . 0027 . 0003 0 . 0219 . 0110 . 0580 . 0189	0. 0120 0005 .0024 0023 .0095 0021 .0027 .0003 0 .0220 .0110 .0580 .0189
$ \begin{cases} (C_2 - C_3)A_1 \\ (C_2 - C_3)A_5 \\ (C_4 - C_{10})A_1 \\ (C_6 - C_{12})A_9 \\ (C_5 - C_{11})A_{11} \\ (C_{10} - C_{10})A_{13} \\ (C_{11} - C_{10})A_{13} \\ (C_{12} - C_{10})A_{13} \\ (C_{12} - C_{10})A_{15} \\ (C_{14} - C_{20})A_{17} \\ (C_{16} - C_{22})A_{19} \\ \sum Z/2 \\ B_3 - \Sigma/2 \\ A_3 = \frac{(B_3 - \Sigma/2)}{P_3} \end{cases} $	-0.1059 2435 1561 .0282 .0020 .0273 0051		. 0824 . 0182 0411 . 0190 . 0092 0201 . 0117 . 0032	. 0823 . 0169 0400 . 0189 . 0087 0197	-0.0499 	0201 0028 0012 0 . 0002 . 0001 . 0001	0200 0026 0011 0 . 0002 . 0001 . 0001 0 0726 0363 3778	$ \begin{array}{c c} \hline (C_{12}-C_{14})A_1, \\ (C_{10}-C_{16})A_3, \\ (C_{3}-C_{16})A_5, \\ (C_{5}-C_{16})A_5, \\ (C_{6}-C_{20})A_7, \\ (C_{4}-C_{22})A_9, \\ (C_{2}-C_{22})A_{11}, \\ (C_{2}-C_{22})A_{13}, \\ (C_{4}-C_{30})A_{17}, \\ (C_{4}-C_{30})A_{19}, \\ \sum \\ Z/2, \\ B_{13}-Z/2, \\ A_{13}= \frac{(B_{13}-Z/2)}{P_{13}} \\ \end{array} $	-0. 0054 . 0273 . 0023 . 0268 1355 2384 2381 1273 . 0255 	2563 . 0824 . 0182 0401 . 0190	2487 . 0823 . 0169 0400	0. 4653 2488 . 0823 . 0169 0400 . 0189 0197 . 0118 . 0032	0.00250070 .0002 .0005 .00560045	-0.00250068 .0002 .0004 .00540045 .00480015 .000100440022 .0300 .0087	-0.00250068 .0002 .0004 .0054004500470015 .000100450022 .0300 .0087
$ \begin{cases} (C_4 - C_5)A_1 \\ (C_2 - C_3)A_2 \\ (C_2 - C_1)A_7 \\ (C_4 - C_1)A_7 \\ (C_5 - C_1)A_1 \\ (C_5 - C_1)A_1 \\ (C_5 - C_1)A_1 \\ (C_1 - C_2)A_1 \\ ($	0. 1584 2435 2361 1356 . 0296 . 0023 . 0245	2563	2487 . 0182 0411 . 0190 . 0092 0201 . 0117 . 0032	2488 . 0169 0400 . 0189		-0. 0736 . 0606 0043 . 0056 . 0006 0 0005 0001 0 0117 0059 . 1567	0041 . 0054 . 0006 0 0005 0001 0 01181	$ \begin{array}{l} (C_{14}-C_{19})A_1, \\ (C_{12}-C_{19})A_3, \\ (C_{19}-C_{29})A_5, \\ (C_{5}-C_{29})A_7, \\ (C_{6}-C_{22})A_9, \\ (C_{4}-C_{29})A_{11}, \\ (C_{2}-C_{29})A_{13}, \\ (C_{2}-C_{29})A_{13}, \\ (C_{2}-C_{23})A_{17}, \\ (C_{4}-C_{34})A_{19}, \\ 2\\ 2\\ B_{15}-2/2, \\ A_{15}=\frac{(B_{15}-2/2)}{P_{15}} \end{array} $	0. 0068 0051 . 0245 . 0021 . 0259 1349 2381 2388 1339 0768 /3. 8373		0. 4650 2487 0823 0400 0189 0032	0. 4653 2488 .0823 .0169 0400 .0189 .0087 .0118 .0032	0.0032 .0013 .0020 00011 0026 0022 	0.0032 .0013 .0020 0 0010 0026 0021 0028 0004 0024 0012 0756 0197	0.0032 .0013 .0020 0 0010 0026 0021 0028 0004 0024 0012 0756 0197
$ \begin{cases} (C_6 - C_8)A_1 \\ (C_4 - C_{10})A_5 \\ (C_2 - C_{12})A_5 \\ (C_2 - C_{12})A_5 \\ (C_3 - C_{12})A_1 \\ (C_4 - C_{13})A_{11} \\ (C_5 - C_{22})A_{13} \\ (C_5 - C_{22})A_{13} \\ (C_{10} - C_{21})A_{17} \\ (C_{12} - C_{20})A_{19} \\ \sum_{[Z](2, -C_{20})} (C_{12} - C_{20})A_{19} \\ \sum_{[Z](2, -C_{20})} (C_{12} - C_{20})A_{19} \\ \sum_{[Z](2, -C_{20})} (C_{12} - C_{20})A_{19} \\ A_{12} = \frac{(B_{11} - \Sigma/2)}{P_{1}} \\ \end{cases} $	0. 0280 1561 2361 2347 1353 . 0268 . 0021 . 0236 0047	2563 . 0824	2487 . 0823 0411 . 0190 . 0092 0201 . 0117 . 0032	2488 . 0823 0400 . 0189 . 0087 0197 . 0118	.0400	0.0097 .0388 0194 .0096 0026 .0002 0 .0003 0 .0366 .0183 .0388	-, 0194 -, 0094 -, 0026 -, 0002 0 -, 0003 0 -, 0364 -, 0182 -, 0389	$ \begin{array}{c} (C_{16}-C_{18})A_1, \\ (C_{14}-C_{20})A_3, \\ (C_{12}-C_{22})A_5, \\ (C_{12}-C_{22})A_7, \\ (C_{16}-C_{20})A_7, \\ (C_{5}-C_{20})A_5, \\ (C_{6}-C_{20})A_1, \\ (C_{6}-C_{20})A_{11}, \\ (C_{7}-C_{20})A_{12}, \\ (C_{7}-C_{20})A_{12}, \\ (C_{7}-C_{20})A_{13}, \\ (C_{7}-C_{7}-C_{7}-C_{7})A_{13}, \\ (C_{7}-C_{7}-C_{7}-C_{7}-C_{7}-C_{7}, \\ (C_{7}-C_{7}-C_{7}-C_{7}-C_{7}-C_{7}-C_{7}, \\ (C_{7}-C_{7}-C_{7}-C_{7}-C_{7}-C_{7}-C_{7}-C_{7}, \\ (C_{7}-C_$	-0.0065 .0040 0053 .0236 .0027 .0262 1273 2388 2380  .0494 /4.2180	2563 . 0824 . 0182 0411 . 0190 . 0092 0201	2487 . 0823 . 0169 0400 . 0189	2488 . 0823 . 0169 0400 . 0189 . 0087 0197 . 0032	-0.0031 0010 0004 .0004 0001 .0005 0012 .0048 0001 0 .0494	-0.0030 0010 0004 .0004 0001 .0005 0011 .0047 0008 0008 0004 .0498 .0118	-0.00300010000400010005001100050011004700080008000400980118
$ \begin{array}{c} (C_3-C_{10})A_1 \\ (C_4-C_{12})A_3 \\ (C_4-C_{14})A_5 \\ (C_2-C_{16})A_7 \\ (C_2-C_{20})A_{11} \\ (C_1-C_{22})A_{13} \\ (C_1-C_{22})A_{13} \\ (C_5-C_{24})A_{15} \\ (C_3-C_{26})A_{17} \\ (C_3-C_{26})A_{19} \\ 2\\ 2\\ 2\\ B_9-\Sigma/2 \\ A_9=\frac{(B_9-\Sigma/2)}{P_9} \end{array} $	1356 2347 2375 1355 . 0259 . 0027	2563 - 0824 - 0182		2488 . 0823 . 0169 . 0189 . 0087 0197 . 0118 . 0032	0072 0112 0043	0112 0040 0045 0012 0005 0 . 0001 0369 0184 1074	0070 0112 0040 0045 0012 0005 0 . 0001 0369 0184 1074	$ \begin{array}{c} (C_{18}-C_{20})A_1 \\ (C_{16}-C_{22})A_3 \\ (C_{14}-C_{24})A_5 \\ (C_{14}-C_{24})A_5 \\ (C_{12}-C_{26})A_7 \\ (C_{10}-C_{29})A_9 \\ (C_{3}-C_{30})A_{11} \\ (C_{6}-C_{32})A_{13} \\ (C_{4}-C_{34})A_{15} \\ (C_{2}-C_{35})A_{17} \\ \Sigma \\ \Sigma/2 \\ B_{19}-\Sigma/2 \\ A_{19}=\frac{(B_{19}-\Sigma/2)}{P_{19}} \end{array} $		. 0190 . 0092 0201 . 0117	. 0823 . 0169 0400 . 0189 . 0087 0197	0. 4653 2488 . 0823 . 0169 04400 . 0189 . 0087 0197 . 0118	0.0017 .0017 .0003 0001 0010 .0002 .0027 0028 .0029 .0015 .0145	0.0017 .0017 .0003 0001 0010 .0002 .0026 0028 .0014 .0146	0.0017 .0017 .0003 .0003 .0001 .0002 .0002 .0028 .0028 .0028 .0014 .0146

### REPORT NO. 585—NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS TABLE VIII.—COMPUTATION OF LOAD DISTRIBUTIONS

	20	18	16	14	12	10	8	6	4	2
θ, degrees	90	81	72	63	54	45	36	27	18	9
Sin \(\theta\). Sin \(\text{36}\). Sin \(\text{56}\). Sin \(\text{56}\). Sin \(\text{56}\). Sin \(\text{90}\). Sin \(\text{110}\). Sin \(\text{130}\). Sin \(\text{150}\). Sin \(\text{150}\). Sin \(\text{150}\). Sin \(\text{150}\).	1. 0000 -1. 0000 1. 0000 -1. 0000 1. 0000 -1. 0000 -1. 0000 -1. 0000 -1. 0000	0.9877 8910 .7071 4540 .1564 .1564 4540 .7071 8910 .9877	0. 9511 5878 0 . 5878 9511 9511 5878 0 . 5878 9511	0. 8910 1564 7071 9877 4540 4540 9877 7071 1564 8910	0. 8090 .3090 -1. 0009 .3090 .8090 8090 3090 1. 0000 3090 8090	0, 7071 .7071 7071 7071 .7071 .7071 .7071 7071 7071 .7071 .7071	0. 5878 . 9511 0 9511 5878 . 5878 . 9511 0 9511 5878	0. 4540 .9877 .7071 1564 8910 8910 1564 .7071 .9877 .4540	0 3090 .8090 1.0000 .8090 .3090 3090 8090 -1.0000 8090 3090	0. 1564 . 4540 . 7071 . 8910 . 9877 . 9877 . 8910 . 7071 . 4540 . 1564
$\begin{cases} A_1 \sin \theta \\ A_2 \sin 3\theta \\ A_3 \sin 3\theta \\ A_4 \sin 5\theta \\ A_7 \sin 7\theta \\ A_8 \sin 9\theta \\ A_{11} \sin 1\theta \\ A_{12} \sin 11\theta \\ A_{13} \sin 15\theta \\ A_{14} \sin 15\theta \\ A_{15} \sin 17\theta \\ A_{16} \sin 17\theta \\ A_{16} \sin 1\theta \\ C_{16} \cos \theta \\ C_$	. 4653 . 2488 . 0823 0169 0400 0189 . 0087 . 01197 . 0118 0032 . 7576 5, 670 4, 296	. 4596 . 2217 . 0382 . 0077 . 0063 . 0030 . 0039 . 0139 . 0105 . 0031 . 7033 6. 152 4. 327	. 4425 . 1462 0 . 0099 . 0381 . 0180 0051 0 . 0069 0030 . 6535 6. 708 4. 384	. 4146 .0389 0582 .0167 .0182 0086 .0086 .0139 0018 .0028 .4451 7. 337 3. 266	.376407690823 .0052032401530027019700370026 .1460 8.029 1.172	. 3290 1759 0582 0120 0283 . 0134 0061 . 0139 . 0084 . 0022 . 0864 8. 772 . 7579	. 2735 2366 0 0161 . 0235 . 0111 . 0083 0 0112 0019 . 0506 9. 520 . 4817	. 2112 2457 . 0582 0026 . 0357 0168 0014 0139 . 0117 . 0014 . 0378 10. 331 . 3905	. 1438 2013 . 0823 . 0137 0124 0058 0070 . 0197 0096 0010 . 0224 12,760 . 2858	
$ \begin{bmatrix} A_1 \sin \theta \\ 3A_3 \sin 3\theta \\ 3A_4 \sin 5\theta \\ 7A_7 \sin 7\theta \\ 9A_9 \sin 9\theta \\ 11A_{11} \sin 11\theta \\ 13A_{12} \sin 13\theta \\ 17A_{17} \sin 17\theta \\ 19A_{19} \sin 19\theta \\ 17A_{17} \sin 17\theta \\ 19A_{19} \sin 19\theta \\ 27A_{17} \sin 1\theta \\ m_*c_3/4b \times \Sigma/\sin \theta \\ c_{d_3} = c_4 \left[ \frac{m_2c_2 \times \Sigma/t}{4b} \right]_{11} $	. 4653 . 7463 . 4115 . 1184 . 3604 . 2078 . 1130 . 2957 . 2009 . 0602 . 14859 . 2852 . 1, 2252	. 4596 . 6650 . 2910 . 0338 0564 . 0325 0513 2091 1790 . 0595 9580 . 1839	. 4425 . 4387 0 . 0696 . 3428 . 1976 0664 0 . 1181 0573 1. 4856 . 2851	.4146 .1167 2910 .1170 .1636 0043 .1116 .2091 0314 .0537 .7696 .1477	. 3764 2306 4115 . 2366 2916 1681 0349 2957 0621 0487 11302 2169 2542	. 3290 5277 2910 0837 2549 0799 0799 2091 1421 0426 0426 0705	. 2735 7098 0 1126 . 2119 . 1221 . 1074 0 1911 0354 3340 0641 0309	.2112 7371 .2910 0185 .3212 1851 0177 2091 .1985 .0273 1183 0227	. 1438 6038 4115 9958 1114 0642 9914 2957 1626 0186 1052 0202 0058	. 0728 3388 . 2910 . 1055 3560 . 2052 . 1007 2091 . 0912 . 0094 0281 0034



Positive directions of axes and angles (forces and moments) are shown by arrows

Axis				ent abo	ut axis	Angle	Э	Velocities		
Designation	Sym- bol	Force (parallel to axis) symbol	Designation	Sym- bol	Positive direction	Designa- tion	Sym- bol	Linear (compo- nent along axis)	Angular	
Longitudinal Lateral Normal	X Y Z	$X \\ Y \\ Z$	Rolling Pitching Yawing	$egin{array}{c} L \\ M \\ N \end{array}$	$\begin{array}{c} Y \longrightarrow Z \\ Z \longrightarrow X \\ X \longrightarrow Y \end{array}$	Roll Pitch Yaw	φ θ ψ	· u v w	$\begin{array}{c c} p \\ q \\ r \end{array}$	

Absolute coefficients of moment

$$C_{l} = \frac{L}{qbS}$$
 (rolling)

$$C_m = \frac{M}{qcS}$$
 (pitching)

$$\frac{vI}{cS}$$

$$C_n = \frac{N}{qbS}$$
 (yawing)

Angle of set of control surface (relative to neutral position),  $\delta$ . (Indicate surface by proper subscript.)

### 4. PROPELLER SYMBOLS

D, Diameter

Geometric pitch

Pitch ratio p/D,

 $V', V_s,$ Inflow velocity

Slipstream velocity

Thrust, absolute coefficient  $C_T = \frac{T}{\rho n^2 D^4}$ T,

Torque, absolute coefficient  $C_Q = \frac{Q}{\rho n^2 D^5}$ Q,

Power, absolute coefficient  $C_P = \frac{P}{\rho n^3 D^5}$ P,

Speed-power coefficient =  $\sqrt[5]{\frac{\rho V^5}{Pn^2}}$  $C_s$ ,

Efficiency

Revolutions per second, r.p.s. n,

Effective helix angle =  $\tan^{-1} \left( \frac{V}{2\pi rn} \right)$ Φ,

### 5. NUMERICAL RELATIONS

1 hp. = 76.04 kg-m/s = 550 ft-lb./sec.

1 metric horsepower = 1.0132 hp.

1 m.p.h. = 0.4470 m.p.s.

1 m.p.s. = 2.2369 m.p.h.

1 lb. = 0.4536 kg.

1 kg = 2.2046 lb.

1 mi. = 1,609.35 m = 5,280 ft.

1 m = 3.2808 ft.